

# Drill-Bit Parity:

The increasing importance of supply-side links in oil and gas markets \*

by

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## Abstract

Previous economic analyses focused on relationships between crude oil and natural gas markets centered primarily on demand-side considerations. We provide evidence that two important supply-side connections play important roles in understanding crude oil and natural gas market integration. First, crude oil production and natural gas production require a common capital input: rotary drilling rigs. Competition for this capital input should be expected to result in comovement in oil prices and gas prices. Second, crude oil wells produce associated gas, while natural gas wells often produce associated oil or oil substitutes. This latter supply-side connection could cause divergence in crude oil prices and natural gas prices, because a price shock for one commodity will increase associated production of the other commodity. Understanding which effect dominates at a particular time is essential to understanding oil and gas market integration. We construct a theoretical model based on the profit-maximizing decisions of a drilling-rig allocating firm to delineate the role played by these supply-side factors in oil and gas market integration. If rig competition is the more important supply-side factor then gas-rig allocations and oil prices will be negatively related, and vice versa, while if associated commodity flows are more important then gas-rig allocations and oil prices will be positively related, and vice versa. We then test the predictions of the theoretical model using three econometric methods, and find ample evidence of long-run relationships driven by these supply-side factors. However, we provide evidence indicating that drilling responses to price changes are unstable across time, and drilling rig competition has come to play a more important role since the onset of increasing production from shale gas and tight oil wells, which began in the mid-2000's.

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# 1 Introduction

Following the financial crisis of 2008 and throughout the subsequent boom in shale oil production, U.S. crude oil and natural gas prices diverged substantially (see Figure 1). This divergence led to analyses indicating the two price series were no longer linked (Erdős, 2012; Ramberg and Parsons, 2012). Past studies generally alluded to demand-side considerations in explanations of the crude oil, natural gas price relationship, because oil and gas are extracted to the same ultimate end: the production of energy (Brown and Yücel, 2008; Hartley et al., 2008). However, supply-driven linkages are plausible as well, as drilling rigs are the primary input necessary for drilling crude oil wells and natural gas wells, so we might expect cost spillovers from drilling wells. Figure 2 displays time series of the employment of gas-directed drilling rigs and oil-directed drilling rigs from January 1997 to June 2016. Further, joint production of natural gas from crude oil wells (sometimes called associated gas) is so commonplace that many analysts believe changes in oil-related drilling activity directly affect natural gas market equilibrium (Wall Street Journal, 2016; Bloomberg, 2015). Similarly, “wet” natural gas wells produce large amounts of liquid hydrocarbons, including oil. These associated commodity flows could cause oil and gas prices to diverge: if a demand-shock increases the price of one of the commodities, the supply of the other commodity would increase through this channel, decreasing its price. Further, the flow of output from each new well is geologically constrained, and supply decisions are highly persistent as a result.

We provide a model of the drilling problem, similar to that of Anderson et al. (2014), that clarifies the importance of each of these features in tying natural gas markets and crude oil markets through the supply side. The theoretical model implies drilling for commodity  $i$  may be negatively or positively related to the price of commodity  $j$  in a supply-side dominated regime, depending

on whether the supply-side relationship is currently dictated by rig competition or by associated commodity flows. We provide empirical evidence that is consistent with steady movement towards a regime dominated by drilling-rig competition beginning in the mid 2000's. This movement is not surprising: production of shale gas and tight oil from onshore reservoirs in the U.S. increased in the mid-to-late 2000's, which increased the cost spillovers from one commodity to the other. At the same time, demand-side substitution, which forms the basis for the demand-side link between oil and gas markets, decreased. Figures 1 and 2 provide anecdotal evidence to this effect. Before 2008, crude oil and natural gas rig allocations tended to move together, while crude oil and natural gas prices also tended to move together, which is consistent with a demand-side dominated relationship. However, crude oil prices recovered and remained elevated relative to natural gas prices after the financial crisis in 2008. This led to a shift of drilling rig allocations from natural gas to oil. Eventually, oil prices decreased dramatically as a result of a worldwide surplus of crude oil production, which can be partially explained by increased drilling in the U.S.

The primary demand-side substitution possibility that has explained the price link between oil and natural gas has declined dramatically since the 1980's: residual fuel oil is the primary oil product that can be substituted with natural gas for electricity generation, and/or space heating, and residual fuel oil supplied to U.S. consumers fell from over a billion barrels per year in 1977 to less than a million barrels in 2015. In 2003, net electricity generation from petroleum liquids was greater than 100,000 Gigawatt hours (GWh), while net generation from natural gas was approximately 650,000 GWh. In 2013, net generation from petroleum liquids had decreased to less than 14,000 GWh, while natural gas generation increased to more than 1.1 Million GWh.<sup>1</sup> Previous analyses of oil and gas price links focused on robust opportunities for demand-side substitution. Such

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<sup>1</sup>See EIA Table 3.1.A. ([http://www.eia.gov/electricity/annual/html/epa\\_03\\_01\\_a.html](http://www.eia.gov/electricity/annual/html/epa_03_01_a.html)).

substitution possibilities were assumed to derive a “burner-tip parity” condition, by which oil and gas prices had a long-run relationship kept in check by cost-minimizing decisions of energy producers (Brown and Yücel, 2008). If producing energy was less expensive using residual fuel oil than natural gas then energy producers would shift consumption toward fuel oil, thus bidding up its price, and vice versa. The decrease in demand-side substitution possibilities, accompanied by the increase in drilling for oil and natural gas in the U.S., motivates our supply-side driven analysis.

Understanding the relationship between oil and natural gas markets is important for several reasons. The shale boom made the U.S. the largest producer of both crude oil and natural gas,<sup>2</sup> and helped dampen the 2007-2009 recession and expedite the subsequent recovery in shale-rich regions (Lim, 2011; Grunewald and Mahon, 2011; Brown and Yücel, 2013). A large literature documents the impact of oil price shocks on the U.S. economy.<sup>3</sup> More recently Hausman and Kellogg (2015) estimate shale-related production of natural gas created an annual increase of \$48 billion in consumer and producer surplus, while Arora and Lieskovsky (2014) find that the impact of natural gas supply on industrial production has increased in the shale era. The increase in production of crude oil and natural gas has also significantly shifted environmental benefits and costs (Johnsen et al., 2016; Knittel et al., 2015; Linn et al., 2014; Muehlenbachs et al., 2013, 2015; Olmstead et al., 2013). Crude oil and natural gas are also responsible for a large share of U.S. emissions of carbon dioxide and many other pollutants. Environmental policies that directly affect one commodity, as the U.S. EPA’s Clean Power Plan will do to natural gas, may indirectly distort the emissions rates of carbon and associated co-pollutants from the other commodity. The extent to which this occurs depends on the mechanism by which the two markets are linked.

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<sup>2</sup>See British Petroleum Statistical Review of World Energy (<http://www.bp.com/en/global/corporate/energy-economics/statistical-review-of-world-energy.html>).

<sup>3</sup>See Balke et al. (2002) and Baumeister and Kilian (2016) for excellent surveys of this literature.

There are three mechanisms that might create a link between crude oil and natural gas markets: demand-side substitution, supply-side rig competition, and supply-side substitution from associated commodity flows. Responses to price shocks will depend on the relative magnitudes of these effects. In a demand-driven regime, a positive oil price shock would lead consumers to shift their consumption toward natural gas, for example by increasing their use of natural gas relative to oil in electricity generation. This in turn would lead to a higher gas price, and increase gas drilling. A positive natural gas price shock would lead a higher oil price, and increased oil drilling in a demand-driven regime for the same reasons. On the other hand, as our theoretical model will show, in a supply-driven regime based on rig competition, a positive oil price shock would lead to increased oil drilling, which would increase the marginal cost of natural gas drilling, and decrease natural gas drilling, which would also lead to a higher gas price. In the absence of associated gas, a positive natural gas price shock would similarly lead to decreased oil drilling, and a higher oil price. On the other hand, when oil wells produce a marketable quantity of natural gas, it is possible that a positive natural gas price shock will lead to increased oil drilling in a supply-driven regime, and a lower oil price. If gas wells produce marketable quantities of oil or close oil substitutes, then a positive oil price shock could lead to increased gas drilling, and a lower gas price. In reality, both demand-side substitution, supply-side competition for drilling rigs, and supply-side substitution driven by associated commodity flows all likely occur simultaneously, so the question of interest is which of these factors dominate at any given time.

Estimating oil-rig responses to gas prices, and gas-rig responses to oil prices, based on our theoretical model, allows us to determine which of these three mechanisms dominates the link between oil markets and gas markets. We test the theoretically derived relationship between rig allocations and prices using three different econometric methods: simple OLS, a vector error correction model,

and three-stage least squares. The latter two methods are employed to check for robustness in the presence of nonstationarity and endogeneity, as rig data and prices are nonstationary, and rig allocations may cause prices to change. Our empirical analysis indicates that a supply-side relationship based on rig competition dominates the relationship between natural gas prices and oil rigs for our full sample. We do not find evidence that a supply-side relationship based on rig competition dominates the relationship between oil prices and gas rigs in the full sample. However, when we reestimate our three models on a rolling 100-month subsample, we find evidence that drilling-rig competition has come to play an increasingly important role for both commodities. Further, evidence is provided that drilling rig allocations are essential in linking natural gas and crude oil prices, as a test of cointegration between the two price series without rig allocations fails to reject the null hypothesis of no cointegration.

## 2 Background

The observed divergence between oil and gas prices starting in 2008 led to a new line of research analyzing the instability of the statistical relationship. See for example Erdős (2012), Ramberg and Parsons (2012), Aloui et al. (2014), Atil et al. (2014), Brigida (2014), Hartley and Medlock III (2014), although instabilities had been documented in some of the earlier literature that established the relationship in the first place, e.g., Villar and Joutz (2006), Brown and Yücel (2008) and Hartley et al. (2008). With the exception of Hartley and Medlock III (2014) who provide a model to clarify and test the role of exchange rates in determining the relative price of crude oil and natural gas this research is almost entirely based on reduced-form time series analysis. Such reduced-form analyses may be acceptable if demand-side substitution were the only mechanism tying together oil and gas

markets. However, such models are not helpful when the demand-driven relationship breaks down. We provide a structural model based on the profit-maximizing decisions of rig-allocating firms to shed light on the supply-side incentives that tie oil and gas markets together.

Figures 3 and 4 provide graphical evidence that rig allocations are responding to relative oil and gas prices, and may be helping to keep the difference between crude oil and natural gas prices in check. The first figure shows monthly natural gas and oil prices per MMBTU from January 2007 to December 2010, and the difference in the number of oil wells and the number of gas wells drilled per month. The latter quantity is negative for the vast majority of the sample period, implying that more natural gas wells were drilled during this period. The widening of the price differential in the favor of oil prices, beginning in early 2009, seems to be accompanied by a large shift toward oil wells. One important piece of information missing from Figure 3 is well productivity. If oil wells are more productive than gas wells on an MMBTU basis, then fewer oil wells need to be drilled to increase oil production relative to natural gas production, and vice versa. For example, suppose the marginal oil well is twice as productive as the marginal gas well. Then, replacing a gas well with an oil well results in a 2 MMBTU increase in oil production per 1 MMBTU decrease in natural gas production, i.e., the slope of the production possibilities frontier in natural gas and oil space is less than negative one. Figure 4 attempts to control for this productivity difference.

Specifically, the *Energy Information Administration Drilling Productivity Report*<sup>4</sup> provides data on new production per rig across several gas and oil producing regions in the U.S. We assume that if a rig is redirected from natural gas to oil, this will happen in the region with the highest oil to natural gas productivity ratio, which is the Bakken in western North Dakota for the entire sample period.

Figure 3 adjusts for this productivity ratio. The productivity ratio varies between a minimum of

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<sup>4</sup>See <https://www.eia.gov/petroleum/drilling/>

7.6 and maximum of 11.4 over the sample period, i.e., an oil well in the Bakken produces between 7.6 and 11.4 times more energy than a natural gas well. Figure 4 shows that once productivity adjustments are made, rig-allocations appear to respond strongly to the price differential providing motivation for a more rigorous economic and statistical analysis of the supply-side relationship between oil and natural gas.

### 3 Model

Consider a representative rig-allocating firm that maximizes a stream of discounted profits by allocating drilling rigs to natural gas reservoirs or crude oil reservoirs. Let the state variables  $z_o(t)$  and  $z_g(t)$  represent the existing flow of crude oil and natural gas from the firm's wells at time  $t$ .<sup>5</sup> At each instant, the firm invests in increasing the flow of each commodity by amounts  $q_o(t)$  and  $q_g(t)$ , which represent the levels of drilling activity, or rig allocations, in crude oil and natural gas reservoirs by the firm at time  $t$ . The history of these rig allocations determines the current flow of output  $z_o(t)$  and  $z_g(t)$ .

The rates of production from previously drilled oil and natural gas wells,  $z_o(t)$  and  $z_g(t)$ , decline over time due to geological characteristics of the underlying reservoir (Cronquist, 2001).<sup>6</sup> For our purposes, we assume that the geological rate of decline is constant across wells and through time within a well, or that there is a constant exponential decline rate, which differs between crude oil

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<sup>5</sup>We adopt notation similar to that of Anderson et al. (2014) for readers familiar with that model.

<sup>6</sup>In Anderson et al. (2014) the state variable is production flow capacity, and actual production flow is a control variable. However, those authors also show that the production flow will be equal to production flow capacity unless there is a steep contango in the commodity market, which is historically rare, so we simplify the model by implicitly assuming that flow capacity is always constrained.

and natural gas.<sup>7</sup> Let  $\alpha_o$  denote the decline rate for oil wells, and let  $\alpha_g$  denote the decline rate for natural gas wells.

We assume the increase in gas flow associated with oil drilling, and the increase in oil flow associated with gas drilling can be characterized by proportions,  $\psi_g$  and  $\psi_o$ , of the firm's respective drilling levels, where  $\psi_g$  and  $\psi_o$  are in  $[0,1)$ . That is, when a firm increases oil flow by  $q_o(t)$  through drilling, it also increases gas flow by  $\psi_g q_o(t)$ , and vice versa. We can now write the dynamics of crude oil and natural gas production from the representative firm's wells as

$$\dot{z}_o(t) = q_o(t) - \alpha_o z_o(t) + \psi_o q_g(t) \tag{1}$$

and

$$\dot{z}_g(t) = q_g(t) - \alpha_g z_g(t) + \psi_g q_o(t). \tag{2}$$

Equations (1) and (2) indicate that production of the commodities increases with drilling levels and decreases due to geological decline. The exponential-decline assumption leads production to decline by a constant fraction of total production. Production of oil and natural gas also increases when the firm drills due to associated commodity flows, which are captured by the last terms on the right-hand sides of (1) and (2).

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<sup>7</sup>We make this simplifying assumption for tractability: natural gas and crude oil wells often exhibit hyperbolic decline, implying that the decline rate decreases as the producing life of the well increases. However, we believe exponential decline offers a reasonable approximation for our purposes.

<sup>8</sup>The firm's total production of gas at  $t$  is

$$z_g(t) = \int_0^t [q_g(s) + \psi_g q_o(s)] e^{-\alpha(t-s)} ds.$$

Using Leibniz Rule to take the time derivative gives

$$\dot{z}_g(t) = -\alpha \int_0^t [q_g(s) + \psi_g q_o(s)] e^{-\alpha(t-s)} ds + q_g(t) + \psi_g q_o(t) = -\alpha z_g(t) + q_g(t) + \psi_g q_o(t).$$

Oil and natural gas wells are drilled into scarce reservoirs. There are a limited number of areas available for drilling, and by choosing to drill an additional well, the firm decreases the number of drilling prospects available. Suppose the stock of drilling prospects available for crude oil at  $t$  is  $A_o(t)$ , while the stock of drilling prospects available for natural gas is  $A_g(t)$ , with initial drilling-prospect stocks  $A_o(0) = A_{o0}$  and  $A_g(0) = A_{g0}$  given exogenously. The dynamics of drilling prospect availability for oil and gas are then

$$\dot{A}_o(t) = -q_o(t) \tag{3}$$

and

$$\dot{A}_g(t) = -q_g(t). \tag{4}$$

It is costly to drill oil and natural gas wells. We assume that drilling costs for each commodity can be broken into two components: costs that are common to drilling both commodities, and costs that are commodity specific. For example, a drilling rig must be allocated whether oil or gas is being drilled for, so the cost of allocating a rig is common to both commodities. Suppose that rigs can be ranked by their desirability for drilling wells, as determined by their vintage or age, power, fuel requirements, transportation cost to the drilling site, etc. — all of which contribute to the cost of getting a unit of crude oil or natural gas into production associated with that rig. If a given rig in the rank order is allocated to one commodity then the next best rig available for drilling the other commodity now comes at a higher cost. We denote the costs that are common to both commodities  $C(q_o(t) + q_g(t))$ , where the argument in this cost function is the sum of drilling levels. We assume these costs are increasing and convex in total drilling intensity.

On the other hand, each commodity also has costs of initiating production that are unique to that commodity. The methods used for hydraulic fracturing and well completion differ between oil and gas. A new gas well may require the construction of new pipelines, whereas oil from a new oil well

can be taken away by truck or other transportation mode which may need to be expanded. The portions of cost that are commodity specific will be denoted  $C_o(q_o(t))$  for crude oil, and  $C_g(q_g(t))$  for natural gas. We similarly assume that these cost functions are increasing and convex in their respective drilling intensities.

The firm receives price  $P_o(t)$  per unit of crude oil produced at time  $t$ , and  $P_g(t)$  per unit of natural gas. The firm will choose crude oil and natural gas drilling intensities through time to maximize the associated discounted stream of profits. The problem faced by the firm, with time dependence suppressed, is

$$\begin{aligned}
& \max_{q_o, q_g} \int_{t=0}^{\infty} [P_o z_o + P_g z_g - C(q_o + q_g) - C_o(q_o) - C_g(q_g)] e^{-rt} \\
& \text{subject to } \dot{z}_o = q_o - \alpha_o z_o + \psi_o q_g, \\
& \dot{z}_g = q_g - \alpha_g z_g + \psi_g q_o, \\
& \dot{A}_o = -q_o, A_{o0} \text{ given}, \\
& \text{and } \dot{A}_g = -q_g, A_{g0} \text{ given},
\end{aligned} \tag{5}$$

where  $r$  is the discount rate applied by the firm. The current-value Hamiltonian associated with (5) is

$$\begin{aligned}
\mathcal{H} = & P_o z_o + P_g z_g - C(q_o + q_g) - C_o(q_o) - C_g(q_g) \\
& - \theta_o q_o - \theta_g q_g \\
& + \mu_o (q_o - \alpha_o z_o + \psi_o q_g) + \mu_g (q_g - \alpha_g z_g + \psi_g q_o) \tag{6}
\end{aligned}$$

The  $\theta_o$  and  $\theta_g$  variables are the shadow values of oil and gas drilling prospects, and represent the opportunity cost associated with using up a scarce drilling prospect at the current instant. The marginal value of production of oil and gas is more than their respective prices in this model, because a marginal unit of flow added in the current instant through drilling represents a stream

of future production. The present values of these production streams at  $t$  are  $\mu_o$  and  $\mu_g$ .

Assuming an interior solution for drilling in both commodities, the first-order conditions associated with (5) are

$$-C'(q_o + q_g) - C'_o(q_o) - \theta_o + \mu_o + \psi_g \mu_g = 0, \quad (7)$$

$$-C'(q_o + q_g) - C'_g(q_g) - \theta_g + \mu_g + \psi_o \mu_o = 0, \quad (8)$$

$$-P_o + \alpha_o \mu_o = \dot{\mu}_o - r \mu_o, \quad (9)$$

$$-P_g + \alpha_g \mu_g = \dot{\mu}_g - r \mu_g, \quad (10)$$

$$\dot{\theta}_o - r \theta_o = 0, \quad (11)$$

and

$$\dot{\theta}_g - r \theta_g = 0. \quad (12)$$

### 3.1 Model Results

In this section, we characterize the optimal drilling decisions of profit-maximizing firms. We derive long-run, inverse-supply loci between the price and the drilling rates for each commodity, as well as inverse-supply loci for flow rates of both commodities. These inverse-supply loci are used to provide intuition surrounding rig-allocation incentives. We then use the inverse-supply loci to derive direct-supply functions with rig allocations on the left-hand side of the equations. These direct-supply equations are less intuitive, but more amenable to empirical analysis, as representative firms are treated as price takers.

Rearranging (7) and (8) to solve for  $\mu_g$  and  $\mu_o$ , we have

$$\mu_o = \frac{(1 - \psi_g)C'(q_o + q_g) + C'_o(q_o) - \psi_g C'_g(q_g) + \theta_o - \psi_g \theta_g}{1 - \psi_g \psi_o}, \quad (13)$$

and

$$\mu_g = \frac{(1 - \psi_o)C'(q_o + q_g) + C'_g(q_g) - \psi_o C'_o(q_o) + \theta_g - \psi_o \theta_o}{1 - \psi_g \psi_o}, \quad (14)$$

which shows that the marginal value of an additional unit of commodity production flow is equal to the marginal cost of initiating an additional unit of flow, plus the shadow value of foregone drilling prospects. Note that associated commodity flows result in cost savings: the firm saves  $\psi_o[C'_o(q_o) + \theta_o]$  in oil costs when it drills gas wells, and vice versa. If the value of flow was higher than the marginal cost plus the shadow value, the firm would allocate additional rigs to the commodity, thus increasing marginal cost for both commodities. The firm would continue to adjust production of each commodity until both (14) and (13) held. Differentiating (14) and (13) with respect to time gives

$$\dot{\mu}_o = \frac{(1 - \psi_g)C''(q_o + q_g)(\dot{q}_o + \dot{q}_g) + C''_o(q_o)\dot{q}_o - \psi_g C''_g(q_g)\dot{q}_g + \dot{\theta}_o - \psi_g \dot{\theta}_g}{1 - \psi_g \psi_o}, \quad (15)$$

and

$$\dot{\mu}_g = \frac{(1 - \psi_o)C''(q_o + q_g)(\dot{q}_o + \dot{q}_g) + C''_g(q_g)\dot{q}_g - \psi_o C''_o(q_o)\dot{q}_o + \dot{\theta}_g - \psi_o \dot{\theta}_o}{1 - \psi_g \psi_o}. \quad (16)$$

Now, we substitute the values of  $\mu_i$  and  $\dot{\mu}_i$  into (9) and (10) and rearrange, so prices are on the left-hand sides. We have

$$P_o = (\alpha_o + r) \left[ \frac{(1 - \psi_g)C'(q_o + q_g) + C'_o(q_o) - \psi_g C'_g(q_g) + \theta_o - \psi_g \theta_g}{1 - \psi_g \psi_o} \right] - \left[ \frac{(1 - \psi_g)C''(q_o + q_g)(\dot{q}_o + \dot{q}_g) + C''_o(q_o)\dot{q}_o - \psi_g C''_g(q_g)\dot{q}_g + \dot{\theta}_o - \psi_g \dot{\theta}_g}{1 - \psi_g \psi_o} \right] \quad (17)$$

and

$$P_g = (\alpha_g + r) \left[ \frac{(1 - \psi_o)C'(q_o + q_g) + C'_g(q_g) - \psi_o C'_o(q_o) + \theta_g - \psi_o \theta_o}{1 - \psi_g \psi_o} \right] - \left[ \frac{(1 - \psi_o)C''(q_o + q_g)(\dot{q}_o + \dot{q}_g) + C''_g(q_g)\dot{q}_g - \psi_o C''_o(q_o)\dot{q}_o + \dot{\theta}_g - \psi_o \dot{\theta}_o}{1 - \psi_g \psi_o} \right]. \quad (18)$$

Equations (17) and (18) say that the firm should allocate rigs in such a way that marginal costs are equal to marginal revenues, or the price received by the firm. The first bracketed terms on the right-hand sides of (17) and (18) represent the cost associated with allocating a marginal rig at instant  $t$ , taking into account subsequent declines and the intertemporal opportunity cost of foregone drilling prospects  $\theta_i$ . These costs are multiplied by  $(\alpha_i + r)$  for the respective commodities, because production will decrease at instantaneous rate  $\alpha_i$  for each commodity, while the present value of the price received will decrease at instantaneous rate  $r$  per period. Therefore, the decision to allocate a rig toward commodity  $i$  can be viewed as a decision to invest in a perpetuity discounted at  $(\alpha_i + r)$ , which is similar in spirit to Kellogg's (2014) model of drilling as an investment.

The firm should also account for costs associated with the rate of change in drilling activity. The incentives related to these costs are captured by the second bracketed terms on the right-hand sides of (17) and (18). Intuitively, the greater the acceleration in drilling, the higher the marginal costs, which is captured by the interactions of second derivatives of the cost functions with the time derivatives of drilling activity. Current marginal costs can temporarily exceed prices if it is intertemporally optimal to accelerate drilling in the current instant, and vice versa. In the steady state, drilling activity is constant, and the second bracketed terms in equations (17) and (18) disappear.

The relationship between commodity prices and drilling levels will depend critically on the respective importance of associated commodity flows, represented by the  $\psi_i$  terms, and the joint cost

function. Large amounts of associated commodity  $i$  flow will increase the incentive to drill wells of commodity  $j$  by saving costs related to drilling type  $i$  wells. A joint cost function will lead the firm to account for cost spillovers related to drilling: an increase in the drilling rate of commodity  $i$  wells will increase the marginal drilling cost of commodity  $j$ , and thus decrease the drilling rate for commodity  $j$ .

Suppose that rig types are completely differentiated by commodity, such that there is no shared cost function for rigs ( $C(q_o + q_g) = 0$ ,  $C_o(q_o) > 0$ , and  $C_g(q_g) > 0$ ). In this case, drilling costs are completely represented by the additively separable cost functions  $C_o(q_o)$  and  $C_g(q_g)$ . Increased drilling for commodity  $i$  will increase the firm's production of commodity  $j$  through the associated-commodity channel. Suppose a positive demand shock increases the price of commodity  $i$ . The firm will increase its drilling rate of commodity  $i$ , which will increase the firm's production of both commodity  $i$ , and commodity  $j$ . If all firms in the market respond in this way, then aggregate production of commodity  $j$  will increase, thus decreasing the price of commodity  $j$ . Therefore, the price increasing demand shock for commodity  $i$  will lead to a lower price for commodity  $j$ . Further, the firm might increase, or decrease its drilling rate for commodity  $j$ : the lower price of commodity  $j$  would lead the firm to decrease drilling, but the higher price of commodity  $i$  will increase the value of the the associated commodity  $i$  flow from commodity  $j$  wells, thus incentivizing increased drilling of commodity  $j$ .

***Prediction 1:*** *In a supply-side regime dominated by associated commodity flows, demand shocks will have an ambiguous effect on drilling rates, and will cause divergence in prices.*

Now suppose there is only a shared cost function for rigs ( $C(q_o + q_g) = 0$  and  $C_o(q_o) = C_g(q_g) = 0$ ), and associated gas and associated oil play insignificant roles ( $\psi_i = 0$ ). This might be true if

no infrastructure is available to collect associated gas from oil wells (in which case all such gas would be "flared"), and if all gas wells were "dry," producing no liquids. Suppose again that a positive demand shock increases the price of commodity  $i$ . The firm will increase the drilling rate of commodity  $i$ , which will increase the marginal cost of drilling commodity  $j$  wells, which will lead the firm to drill fewer commodity  $j$  wells. If all firms in the market respond in this way, then aggregate production of commodity  $j$  will decrease, and the price of commodity  $j$  will increase. In this case, the price increasing demand shock for commodity  $i$  will lead to less commodity  $j$  drilling, and a higher commodity  $j$  price. These two cases highlight the importance of understanding the supply-side mechanisms connecting oil and gas markets, as they lead to opposite conclusions concerning the relationship between oil and gas prices.

***Prediction 2:*** *In a supply-side regime dominated by drilling-rig competition, demand shocks will cause divergence in drilling rates, and co-movement in prices.*

In order to simplify equations (17) and (18), and further clarify the roles played by associated commodity flows and a joint drilling cost function, we make several simplifying assumptions. First, we assume the cost functions all take quadratic forms, so that the marginal costs are linear in drilling levels. That is, we write the marginal costs as

$$C'(q_o + q_g) = b \cdot (q_o + q_g), \quad (19)$$

$$C'_o(q_o) = b_o q_o, \quad (20)$$

and

$$C'_g(q_g) = b_g q_g, \quad (21)$$

where  $b$ ,  $b_o$ , and  $b_g$  are constants. This allows well-level productivity to differ across oil and natural gas wells, but implies that well-level productivities are constant across time, which is a somewhat

restrictive assumption.<sup>9</sup> We also assume the shadow values of drilling prospects are constant, which may be the case if new drilling prospect discoveries or new demand-side technologies temper the growth of drilling-prospect value associated with scarcity. This amounts to assuming that drilling prospects are not viewed as being exhaustible by the representative firm, or that firms believe new discoveries and/or technological progress temper shadow value growth, so a long-run equilibrium with stable rig allocations exists. Previous empirical research is mixed on intertemporal changes in oil shadow values. Adelman (1993) found that the oil shadow value tended to be constant over time, while Anderson et al. (2014) find that oil drilling prospect shadow values increased in Texas over their sample period.<sup>10</sup> If the shadow value of drilling prospects is constant, then  $\dot{q}_o^* = \dot{q}_g^* = \dot{z}_o^* = \dot{z}_g^* = 0$  in long-run equilibrium, because firms cannot profit by inter-temporally reallocating drilling rigs or production, and drilling of new wells exactly balances the decline of production from existing wells. Using these assumptions, we can rewrite (17) and (18) as

$$P_o = \frac{\alpha_o + r}{1 - \psi_g \psi_o} \{[(1 - \psi_g)b + b_o] q_o + [(1 - \psi_g)b - \psi_g b_g] q_g + \theta_o - \psi_g \theta_g\} \quad (22)$$

and

$$P_g = \frac{\alpha_g + r}{1 - \psi_g \psi_o} \{[(1 - \psi_o)b + b_g] q_g + [(1 - \psi_o)b - \psi_o b_o] q_o + \theta_g - \psi_o \theta_o\}. \quad (23)$$

Equations (22) and (23) represent steady-state drilling-price loci for each commodity given our simplifying assumptions. They can be used to derive steady-state, inverse-supply curves for crude oil

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<sup>9</sup>We investigate whether these parameters change in the empirical section by estimating the parameters on sub-samples.

<sup>10</sup>The question of drilling prospect shadow value growth is not central to our research question. To the extent that this assumption is violated it will affect the quantitative results, but not the qualitative results of our empirical analysis below. Further, Anderson et al. (2014) focus on state-level data, while we focus on U.S.-level data. The assumption of constant shadow values is more likely to hold at higher levels of aggregation, because the possibility of new discoveries, and the number of drilling prospects to which new technologies might be applied, are both larger in the U.S. than in Texas.

and natural gas by plugging in the steady-state conditions for constant commodity flow. Equations (1) and (2) can be used to write

$$\dot{z}_o = 0 \implies q_o = \alpha_o z_o - \psi_o q_g, \quad (24)$$

and

$$\dot{z}_g = 0 \implies q_g = \alpha_g z_g - \psi_g q_o. \quad (25)$$

We can substitute  $q_g$  out of (24) and substitute  $q_o$  out of (25) to get the steady-state drilling rates, which are

$$q_o = \frac{\alpha_o z_o - \psi_o \alpha_g z_g}{1 - \psi_o \psi_g}, \quad (26)$$

and

$$q_g = \frac{\alpha_g z_g - \psi_g \alpha_o z_o}{1 - \psi_o \psi_g}. \quad (27)$$

The steady-state drilling rates are larger when the own decline rates (i.e.,  $\alpha_o$  for  $q_o$  and  $\alpha_g$  for  $q_g$ ) are larger, as more production flow is lost at each instant due to geological decline. The steady-state drilling rates are also larger when associated commodity flow is larger, i.e.,  $q_o$  is larger when  $\psi_g$  is larger, and vice versa, because if  $\psi_i$  is higher, then less  $q_i$  is needed to make of for production decreases related to geological decline. The steady-state flow rates can be used to derive the steady-state inverse-supply curve by substituting  $q_o$  and  $q_g$  out of (22) and (23), which gives

$$\begin{aligned} P_o = & \frac{\alpha_o(\alpha_o + r)}{(1 - \psi_o \psi_g)^2} [(1 - \psi_g)^2 b + b_o + \psi_g^2 b_g] z_o \\ & + \frac{\alpha_g(\alpha_o + r)}{(1 - \psi_o \psi_g)^2} [(1 - \psi_g)(1 - \psi_o)b - \psi_g b_g - \psi_o b_o] z_g \\ & + \frac{\alpha_o + r}{1 - \psi_g \psi_o} (\theta_o - \psi_g b_g), \quad (28) \end{aligned}$$

and

$$\begin{aligned}
P_g = & \frac{\alpha_g(\alpha_g + r)}{(1 - \psi_o\psi_g)^2} [(1 - \psi_o)^2b + b_g + \psi_o^2b_o] z_g \\
& + \frac{\alpha_o(\alpha_g + r)}{(1 - \psi_o\psi_g)^2} [(1 - \psi_g)(1 - \psi_o)b - \psi_o b_o - \psi_g b_g] z_o \\
& + \frac{\alpha_g + r}{1 - \psi_g\psi_o} (\theta_g - \psi_o b_o). \quad (29)
\end{aligned}$$

We can see that the inverse-supply curves are steeper (i.e.,  $\frac{dP_i}{dz_i}$  is larger) not only when cost parameters are larger, but also when decline rates and interest rates are larger. When interest rates are higher, the cost of dedicating financial capital to drilling is higher, while higher decline rates imply less total production from wells. A larger associated gas parameter,  $\psi_g$ , has an ambiguous effect on the slope of the inverse-oil-supply curve, and similarly a larger associated oil parameter,  $\psi_o$ , has an ambiguous effect on the slope of the inverse-gas-supply curve. These effects depend on the relative magnitudes of the cost parameters  $b$ ,  $b_g$ , and  $b_o$ , and also on the magnitude of the opposite associated commodity parameter. Shifts in the supply curves occur when supply of the opposite commodity changes or the scarcity of drilling prospects, reflected in  $\theta_i$ , changes, and these shifts are larger when decline rates and interest rates are greater. Replacing the left hand side of equations (28) and (29) with inverse demand equations for each commodity would result in a system of two equations that implicitly define the steady-state market equilibrium commodity flow rate for the two commodities.

The relationships derived above, with prices on the left-hand sides were useful for explaining supply-side connections between crude oil markets and natural gas markets. However, direct supply-curves are more amenable to empirical analysis, because while crude oil prices and natural gas prices play a primary role in the rig-allocation decisions of firms, many factors other than rig allocations will affect natural gas and crude oil prices. We can rearrange (22) and (23) to derive curves relating

drilling rates to natural gas prices and crude oil prices. First, let

$$\Gamma_o = (1 - \psi_g \psi_o) / [(1 - \psi_g)(\psi_g - \psi_o)b^2 + (1 - \psi_g \psi_o)(b + b_g)b_o + (1 - \psi_g^2)bb_g], \quad (30)$$

and

$$\Gamma_g = (1 - \psi_g \psi_o) / [(1 - \psi_o)(\psi_o - \psi_g)b^2 + (1 - \psi_g \psi_o)(b + b_o)b_g + (1 - \psi_o^2)bb_o]. \quad (31)$$

Note that  $\Gamma_o$  will be positive unless the associated gas parameter,  $\psi_g$ , is very small relative to the associated oil parameter,  $\psi_o$ , while  $\Gamma_g$  will be positive unless  $\psi_o$  is very small relative to  $\psi_g$ .

Rearranging (22) and (23), and using  $\Gamma_o$  and  $\Gamma_g$ , we can write

$$\begin{aligned} q_o &= \Gamma_o \left\{ \frac{(1 - \psi_o)b + b_g}{\alpha_o + r} P_o + \frac{\psi_g b_g - (1 - \psi_g)b}{\alpha_g + r} P_g + (\theta_g - \theta_o)b - \theta_o b_g \right\} \\ &= \beta_{oo} P_o + \beta_{og} P_g + \beta_{o0}, \end{aligned} \quad (32)$$

and

$$\begin{aligned} q_g &= \Gamma_g \left\{ \frac{(1 - \psi_g)b + b_o}{\alpha_g + r} P_g + \frac{\psi_o b_o - (1 - \psi_o)b}{\alpha_o + r} P_o + (\theta_o - \theta_g)b - \theta_g b_o \right\} \\ &= \beta_{gg} P_o + \beta_{go} P_g + \beta_{g0}. \end{aligned} \quad (33)$$

As expected, an increase in the oil price will lead to more oil drilling, and an increase in the gas price will lead to more gas drilling, as can be seen from the first terms inside the brackets in equations (32) and (33).

This representation provides another way to contrast a supply-side relationship to a demand-side relationship, and reiterate predictions (1) and (2). In a demand-side relationship characterized by substitutability between oil and gas, a demand shock that raises the price of one commodity will cause an outward demand shift in the other commodity that raises both its price and quantity

produced. Here we see that in a supply-side relationship, the same demand-driven price increase in one commodity will cause an ambiguous shift in supply of the other commodity. For example, consider a positive demand shock that increases the price of natural gas. From the second term in the brackets in (32) we can see that if the associated gas parameter,  $\psi_g$ , and/or the gas-specific marginal cost slope,  $b_g$ , are large relative to the shared marginal cost slope parameter,  $b$ , then the natural gas price shock will increase the supply of oil. Intuitively, more oil drilling will occur because a relatively large amount of associated natural gas will be produced from oil wells, a relatively large amount of money will be saved from the gas-specific marginal cost, and cost spillovers from rig employment will be relatively low. On the other hand, if the shared marginal cost slope parameter is large, and the associated gas and gas-specific marginal cost slope parameters are low, then the natural gas price shock will lead to a decrease in oil drilling. In the latter case, oil drilling significantly increases the marginal cost of gas drilling, so oil drilling will be decreased, and replaced with gas drilling. Analogous results hold for natural gas drilling responses to oil price shocks. Which of these cases prevails is an empirical question, which we explore in the next section.

## 4 Empirical Analysis

In this section, we empirically test the predictions of our rig-allocation model. We focus our empirical analysis on the equations with rig allocations on the left-hand side, or equations (32) and (33). We focus on these equations, because we are interested in the rig allocation problem as a response to commodity prices, and prices are affected by factors such as international crude oil production and weather in ways unrelated to rig allocations. Also, in the U.S. marginal rig allocating firms are likely price-takers.

Three empirical methods are implemented. Equations (32) and (33) are tested using simple OLS, a vector error correction model (VECM), and a simultaneous-equations model (3SLS). The latter two models estimate (32) and (33) simultaneously, and are implemented to check for robustness of the OLS estimates related to nonstationarity and endogeneity, respectively. Nonstationarity in the data might lead to spurious results in the OLS model, while endogeneity in the prices may lead to biased estimates. However, we find that all three methods lead to qualitatively similar results. We investigate stability of the coefficient estimates over time by reestimating each model on a rolling 100-month subsample. Results indicate that a supply-side relationship in which the shared marginal cost slope parameter is large relative to the associated gas and gas-specific marginal cost slope parameters dominate the oil drilling/price relationship for the full sample, and this relationship has become stronger over time: All three empirical methods give statistically and economically significant negative gas price coefficients in the estimation of (32). On the other hand, the estimated oil price coefficient in the natural gas drilling/price relationship is small and positive under the OLS model, and insignificantly different from zero in both the VECM and 3SLS model.

If the variables of interest are non-stationary, then simple OLS may give spurious results, unless the residuals are stationary, in which case the series are cointegrated (Hamilton, 1994). In order to check the robustness of the OLS results, we estimate equations (32) and (33) using Johansen's maximum likelihood procedure for cointegrated systems, which is robust to nonstationarity in the levels of our data series (Johansen, 1988). Equations (32) and (33) will be the cointegrating vectors in a VECM, which is estimated in the series' differences. The optimal number of lags for the VECM is selected using a variety of information criteria. The final prediction error (FPE), Akaike's information criterion (AIC), Hannan and Quinn's information criterion (HQIC), and Schwarz's

Bayesian information criterion (SBIC) all indicate that the optimal number of lags in the underlying VAR of the system is two, so we use two lags in our VECM estimation. Johansen's trace statistic test indicates that there are no more than two cointegrating vectors in the system including oil rigs, gas rigs, oil prices, and gas prices, so we simultaneously estimate two cointegrating vectors based on (32) and (33) in the VECM. We also tested for the existence of a direct cointegrating vector between oil prices and gas prices without including rig allocations, again using Johansen's trace statistic. This test failed to reject the null hypothesis of zero cointegrating vectors, suggesting that rig-allocations play a key role in crude oil and natural gas market integration. Our results are qualitatively similar across the OLS and VECM methods. The primary difference is that the OLS model gives a small, but statistically significant, oil-price coefficient in the estimation of the direct natural gas supply curve, (33), while the analogous coefficient in the VECM is insignificantly different from zero.

A simultaneity problem may exist, because rig allocations are determined in market equilibrium with prices, so using simple OLS to estimate (32) and (33) may give biased estimates. We choose to use spot oil and gas prices in the OLS regressions to partially alleviate this problem. Rig allocations in month  $t$  will not affect oil and natural gas supply until later than  $t$ , as it takes time for wells to begin producing oil and gas after drilling. Therefore, we would not expect rig allocations in month  $t$  to directly affect spot prices in month  $t$ . However, rig allocations in month  $t$  may affect spot prices in month  $t$  through less direct channels. For example, rig allocations at  $t$  will affect expectations about future supply, and thus futures prices, which in turn should be expected to affect spot prices through storage incentives. If natural gas futures prices increase because natural gas rig allocations decrease, then natural gas storage operators should be expected to buy spot natural gas and put it into storage, thus bidding up spot prices. This motivates an instrumental-variable robustness

check.

Roberts and Schlenker (2013) suggest using lagged shocks to storage levels to identify supply and demand parameters for storable commodities. Hausman and Kellogg (2015) implement such a strategy for natural gas using weather data, as weather will cause exogenous shocks to natural gas storage levels – natural gas is used for home heating in the winter, and is used for electricity generation during the summer. In other words, weather exogenously shifts demand for natural gas, and can be used to identify supply parameters. We adopt a similar strategy to identify our natural gas price parameters using deviations from normal population-weighted cooling-degree days and heating-degree days.<sup>11</sup> We use a single-month lag as well as a sum of 12 single-month lags as instruments in our 3SLS model. The sums capture the cumulative affect of weather shocks on storage levels (Hausman and Kellogg, 2015). In order to identify the crude oil-price parameters, we would like a similar exogenous demand shifter. Demand for crude oil in the U.S. primarily comes from refineries that produce refined-petroleum products such as gasoline from crude oil. We use a simple VAR to forecast refinery inputs (primarily crude oil). We assume errors from this forecast represent unexpected changes in crude oil demand, and construct an instrument using a sum of 12 lags of these errors as our crude-oil demand shifter. Unexpected changes in refinery inputs will shock storage levels, so this strategy is similar to that suggested by Roberts and Schlenker (2013). We also include lagged errors from simple natural gas price and crude oil price VARs using three lags. Lagged errors from such VAR’s capture price shocks related to other unobservable factors that shift the residual demand curve faced by rig allocating firms. Our 3SLS estimates are reasonably consistent with our OLS estimates. The only divergence, again, is the oil-price coefficient in the estimation of (33). This coefficient is insignificantly different from zero in the 3SLS model.

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<sup>11</sup>A heating-degree day measures the difference in the average daily temperature from 65 degrees if the temperature is below 65 degrees, while heating-degree days are zero if the average daily temperature is above 65 degrees.

After showing that the estimated parameters are qualitatively similar across the OLS, VECM, and 3SLS models. We roll each estimation procedure over our sample using a window of 100 observations. This allows us to investigate if and how the parameters of the supply curves (32) and (33) have changed over time.

## 4.1 Data

The data for this analysis are taken from the *Energy Information Administration (EIA)*, the *National Oceanic and Atmospheric Administration (NOAA)*, and the *Bureau of Labor Statistics (BLS)* websites.<sup>12</sup> The time series used for our empirical analysis include: crude oil prices, natural gas prices, crude oil rig levels, natural gas rig levels, gross inputs to atmospheric crude oil distillation units, deviations from normal heating-degree days, deviations from normal cooling-degree days, and the producer price index for all commodities.

We measure our left-hand side variables of interest, drilling levels, using EIA “rotary rigs in operation data,” whose original source is Baker Hughes. The rig data are broken into several categories including “crude oil directed rigs” and “natural gas directed rigs.” The rotary rigs in operation dataset is limited to monthly granularity, so we use monthly observations. The range of the full-estimation sample is January 1997 to June 2016.

The price data are spot prices.<sup>13</sup> The crude oil price is the West Texas Intermediate price (WTI) measured in dollars per barrel, which is the U.S. benchmark price for oil, while the natural gas

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<sup>12</sup>[www.eia.gov](http://www.eia.gov), [www.noaa.gov](http://www.noaa.gov), and [www.bls.gov](http://www.bls.gov).

<sup>13</sup>Estimation was also performed with prompt month futures and four-month futures for each commodity, and the results were very similar. Results from that analysis are available upon request.

price series is the Henry Hub price (HH) measured in dollars per MMBTU, which is the U.S. benchmark price for natural gas. These price series are inflated to April 2016 U.S. dollars using the BLS Producer Price Index (PPI) for all commodities. We chose the PPI rather than the CPI, as we are considering the decisions of producers. The oil price series is divided by six, as we assume there are six MMBTU per barrel of oil. This has no effect on the results and is done for ease of results interpretation, especially when we roll over the sample and present the rolling results in charts.

Deviations from population-weighted heating and cooling-degree data measure differences from thirty-year average population-weighted heating and cooling-degree days, and are used as an exogenous measure of natural gas demand. Heating- and cooling-degree days measure temperature deviations from 65 degrees Fahrenheit. Cooling degree days are the analogous measure for temperatures above 65 degrees. NOAA adds weights to heating and cooling-degree day observations using census population data. These degree days are summed to get monthly observations, then these monthly observations are differenced from thirty-year averages to get deviations from normal. It is important to use deviations from normal, because natural gas market stakeholders such as traders and storage operators will form expectations about weather, and prepare for average weather. Deviations from average weather represent surprises.

Lastly, the gross inputs to atmospheric crude oil distillation units are used as a measure of crude oil demand. This measure is of course not purely exogenous, so we use a simple VAR forecasting model that regresses gross inputs on three lags of gross inputs. We assume the errors of this forecasting model represent exogenous surprises to crude oil demand.

## 5 Empirical Results

Table 1 presents results from our three estimation strategies on the full sample. The upper half of the table presents results from the estimation of equation (32), while the lower half of the table presents results of the estimation of (33). The first column of Table 1, labeled “*OLS*,” shows results from ordinary least squares estimation, the second column, labeled “*VECM*,” contains results from Johansen’s maximum likelihood estimation of a cointegrated system, while the third column, labeled “*3SLS*,” contains results from the three-stage least squares procedure.<sup>14</sup>

First, the own-price effects are all positive, statistically significant, and economically significant. These estimated coefficients indicate that a \$1.00 per MMBTU increase in the price of oil, or equivalently a \$6.00 per barrel increase in the price of oil, led to an increase of between 50 and 100 oil-directed rigs on average over the sample. Similarly, a \$1.00 per MMBTU increase in the price of natural gas led to an increase of between approximately 65 and 110 gas-directed rigs on average over the full sample.

Second, the estimated coefficients on gas price in the oil-rig regressions are all negative, statistically significant, and economically significant. These estimated coefficients indicate that a \$1.00 per MMBTU increase in the price of natural gas led to a decrease of between 65 and 165 oil rigs on average over the full sample period. On the other hand, the estimated coefficients on oil price in

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<sup>14</sup>The short-run parameters from the VECM estimations, and some alternative instrument specifications for the 3SLS model are presented in the appendix. The short run parameters from the VECM model reinforce our assumption that U.S. rigs respond to prices, while prices do not respond robustly to rig counts, as the parameters indicate that lagged differences of oil prices statistically significantly impact differences in oil rigs, and both lagged differences of oil prices and gas prices statistically significantly impact differences in gas rigs.

the natural gas rig regression are all small in magnitude. They also change sign: the estimate from OLS is positive and statistically significant. It indicates that a \$1.00 per MMBTU increase in the oil price resulted in an average increase of approximately 17 natural gas rigs in the full sample. On the other hand, the estimates from the Johansen procedure and 3SLS are negative, but not significantly different from zero.

The results from the oil-drilling regression, which represents our estimates of equation (32) indicate that a supply-side relationship associated with drilling-rig competition plays an important role in the allocation of oil-directed drilling rigs. The negative and significant coefficients on gas price indicate that as gas prices rise, fewer oil-directed rigs are employed, which is consistent with the shared cost function dominating the effects of associated natural gas and the gas-specific cost function. On the other hand, the results from the gas-drilling regression, which represents our estimates of equation (33) indicate that the associated oil and/or the oil-specific cost function play more important roles in the allocation of gas-directed rigs than the shared cost function. This result is likely associated with a large number of “wet” natural gas wells, which produce large quantities of liquid hydrocarbons including crude oil.

Figures 5 and 6 show how the own- and cross-price effects change over time as we roll OLS estimation over the sample using sample windows of 100 observations. The labels on the horizontal axis indicate the month of the beginning of the subsample. Figures 7 and 8 show the analogous rolling estimates from the VECM, while Figures 11 and 12 show the rolling estimates from the 3SLS model. Due to the large outliers from the VECM rolling procedure displayed in Figures 7 and 8, we also include charts in Figures 9 and 10 showing the rolling VECM estimates for the sample starting only after November 2004, after which no outlying estimates occur.<sup>15</sup> Figures 9 and 10 show cross-price

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<sup>15</sup>The outlying coefficients in Figures 7 and 8 occur in subsamples of size 100 that start around 1998, and then again

effects are negative and stable for both equations, which provides evidence that rig-responses in both equations 32 and 33 settled into a robust and relatively stable supply-driven relationship in which a shared cost function played an important role.

All of the figures representing our rolling estimates indicate cross-price coefficients become smaller, and eventually negative for all three econometric methods. This is consistent with the supply-side relationship associated with the shared cost function becoming more important in the most recent subsamples. In the rolling OLS and VECM charts, cross-price effects remain negative through the end of the sample, while in the 3SLS model the coefficient on gas price in the oil drilling equation remains negative until the end of the sample, but the coefficient on oil price in the gas rig equation eventually increases and becomes positive again. This last fact may appear anomalous, but could indicate that the shared cost function has become less important as natural gas and oil prices have fallen to very low levels recently, and rig employment for both commodities has decreased substantially as a result, as indicated in Figures 1 and 2.

The instability of our estimates of the parameters in equations (32) and (33) illustrates the importance of accounting for supply-side factors in any study of crude oil market and natural gas market integration. The relationship between these two commodity markets, and the relationship between crude oil prices and natural gas prices will depend critically on the current nature of the for subsamples beginning between 2003 and 2005. We believe the outliers in the 1998 subsamples are associated with the outsized effect of an extremely cold December in 2000, as measured by deviations from normal population-heating degree days, which led to extremely low natural gas storage levels, spiking natural gas prices, and a large natural gas rig response. The natural gas price spike and associated rig responses can be seen in Figures 1 and 2. Many anomalies might explain the outliers appearing for samples starting between 2003 and 2005, including the housing bubble and following housing market crash, but also the application of new technologies associated with shale gas and tight oil, which vastly increased the availability of drilling prospects first for natural gas, and then for crude oil.

supply-side connections.

## 6 Conclusion

In this paper, we constructed a model accounting for important supply-side characteristics that tie together crude oil and natural gas markets. We argued that the existence of drilling rigs as a common capital input in natural gas and crude oil production could lead to cost spillovers, so increased drilling for crude oil could plausibly increase the cost of drilling natural gas wells and vice versa. Therefore, increased drilling of one type of commodity should be expected to be associated with decreased drilling of the other type of commodity. This type of supply-side integration would tend to keep the difference in natural gas prices and oil prices in check. However, we also incorporated associated commodity flows into our analysis. Associated commodity flows occur when gas wells produce oil, and/or oil wells produce gas. We showed that if associated commodity flows dominate supply-side integration of crude oil and natural gas markets, then prices may move apart. Thus, understanding the factors that currently dominate supply-side integration in crude oil and natural gas markets is essential in any analysis of the relationship between crude oil and natural gas prices.

Our empirical results indicated that drilling-rig competition has dominated the relationship between oil rig allocations and natural gas prices for our full sample, because we found that higher gas prices tend to be associated with lower oil rig allocations. We also found that natural gas rigs did not respond in an economically significant way to oil prices when using our full sample. Further, we showed that the effect of oil and gas prices on oil and gas rig allocations changed substantially

over our sample by rolling estimation strategies over the sample. These changes could occur for two reasons that we explained using our theoretical model: changes in the scarcity of drilling rigs, and changes in associated commodity flows. Associated commodity flows could change if more or less natural gas flowing from oil wells is flared (which could occur if infrastructure to take away associated gas is built), if more oil wells are drilled in areas without preexisting natural gas takeaway capacity, or if the balance of “wet” and “dry” natural gas wells changes. All of these factors will affect the responses of rig allocations to price changes, which in turn will affect oil and gas market integration.

The importance of understanding the relationship between crude oil and natural gas prices cannot be overstated. Crude oil and natural gas production have become increasingly important to the U.S. economy ever since new technologies associated with shale gas and tight oil have unlocked massive new domestic oil and gas resources. Further, the current state of natural gas markets and crude oil markets has large environmental implications. Increasing use of natural gas in electricity generation has displaced coal, and led to environmental benefits. However, low gas prices could lead to increased crude oil production, and lower oil prices, which could have negative environmental impacts. This study highlights the important role of supply-side considerations in our understanding of crude oil market and natural gas market integration.

## 7 Tables and Figures

Figure 1: *Monthly spot prices of crude oil and natural gas. Source: EIA*

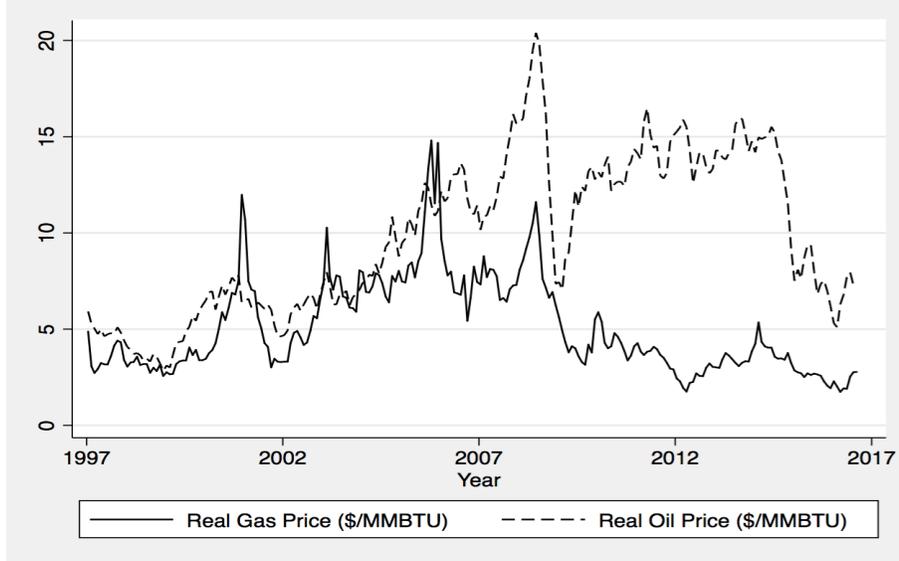


Figure 2: *Monthly crude oil directed rigs and natural gas directed rigs. Source: EIA*

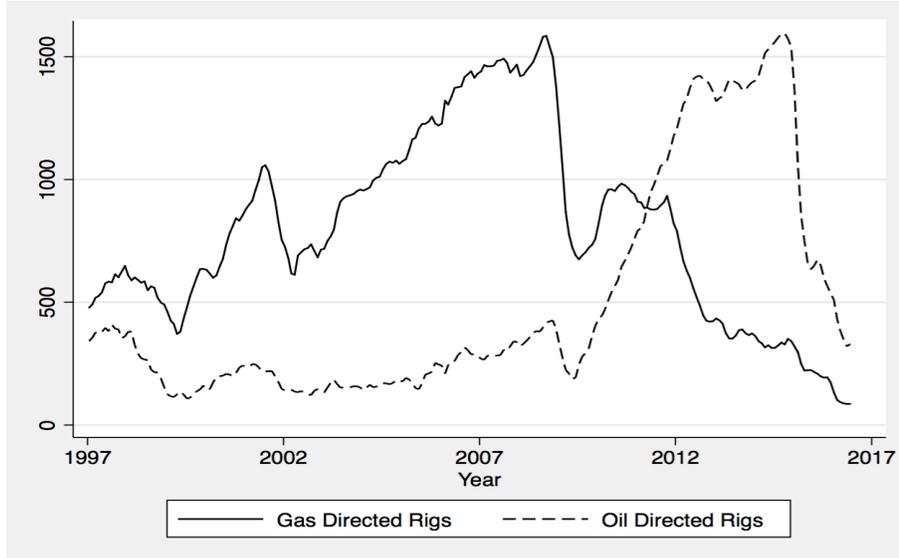


Figure 3: The difference in the number of oil wells and gas wells drilled by month, with corresponding oil and gas prices.

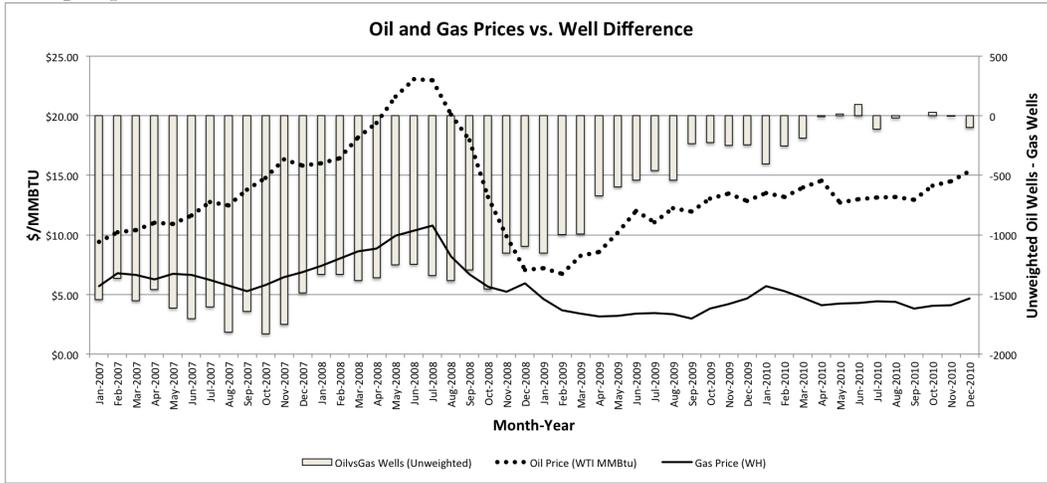


Figure 4: The difference in the number of energy and productivity weighted oil wells and gas wells drilled by month, with corresponding oil and gas prices.

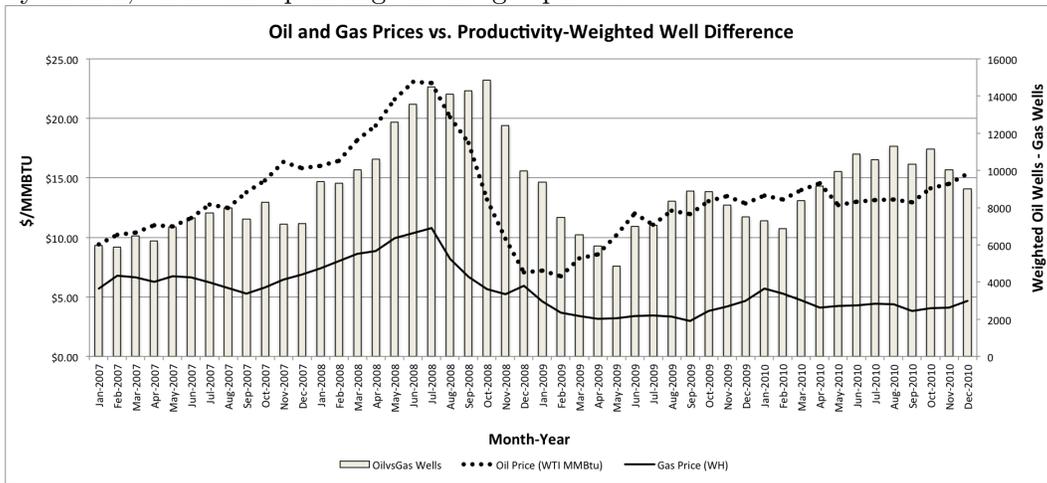


Table 1: Full-sample estimates of rig responses to prices.

	(1)	(2)	(3)
	<i>OLS</i>	<i>VECM</i>	<i>3SLS</i>
$\mathbf{q}_o(t)$			
$P_g(t)$	-101.67*** (7.38)	-163.56*** (26.43)	-67.00*** (16.85)
$P_o(t)$	75.39*** (4.48)	104.50*** (15.63)	52.17*** (12.22)
cons	282.93*** (54.86)	318.01 (183.66)	333.93** (132.89)
$R^2$	0.63	0.48	0.57
$\mathbf{q}_g(t)$			
$P_g(t)$	107.75*** (6.64)	175.93*** (21.58)	64.78*** (16.12)
$P_o(t)$	17.26*** (4.04)	-5.90 (12.32)	-1.41 (11.69)
Constant	76.77 (49.41)	-60.02 (152.29)	484.01 (127.14)
$R^2$	0.58	0.37	0.44
$N$	234	232	229

Standard errors in parentheses

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$

Figure 5: Coefficients from rolling oil rig OLS regression over sample (100 observation windows).

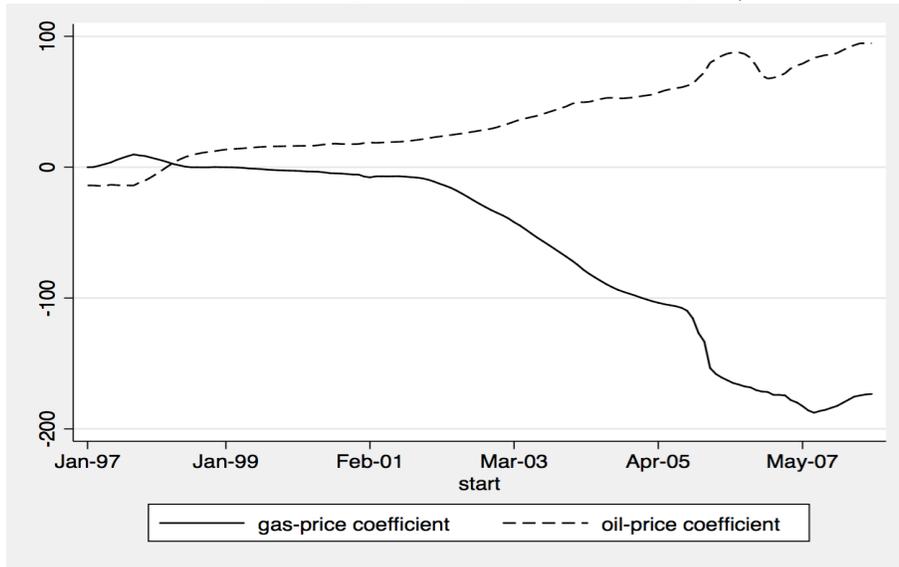


Figure 6: Coefficients from rolling gas rig OLS regression over sample (100 observation windows).

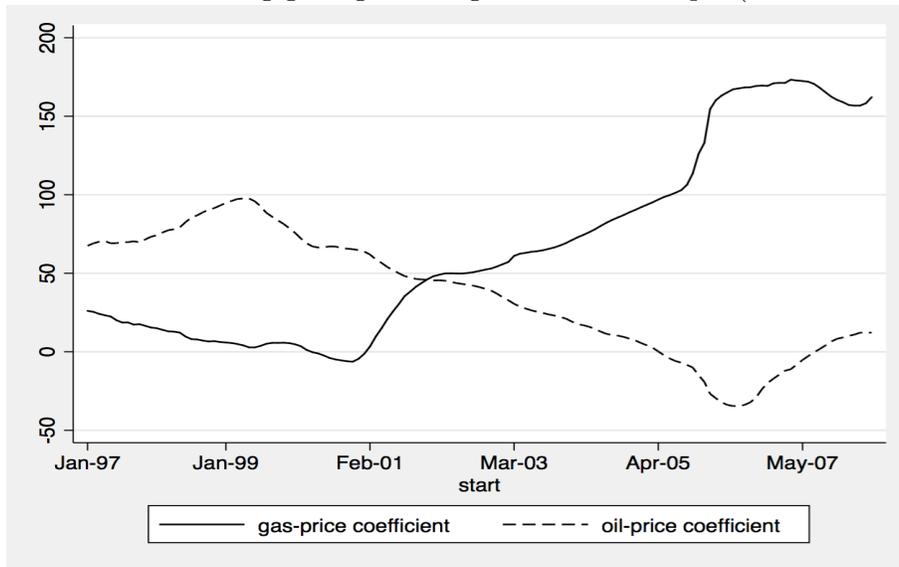


Figure 7: Coefficients from rolling oil rig cointegrating equation over sample (100 observation windows).

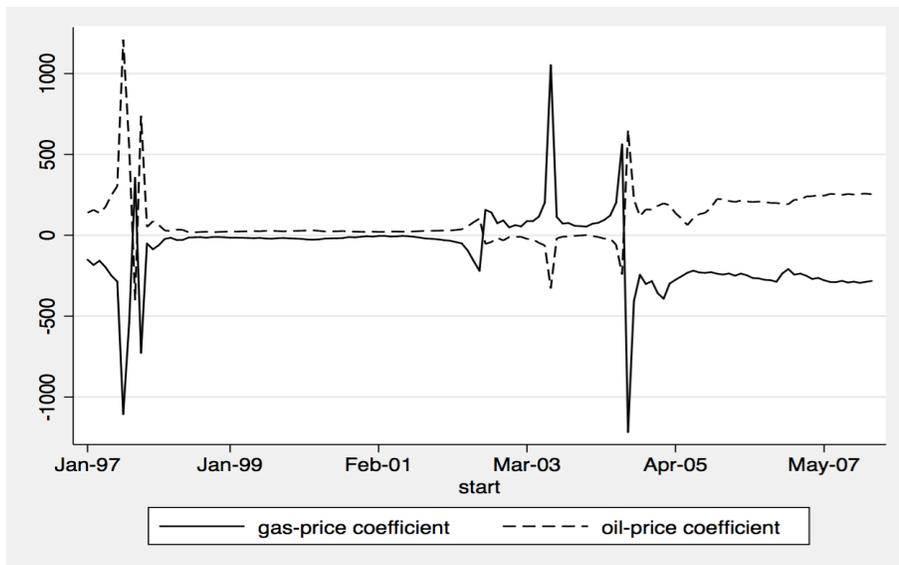


Figure 8: Coefficients from rolling gas rig cointegrating equation over sample (100 observation windows).

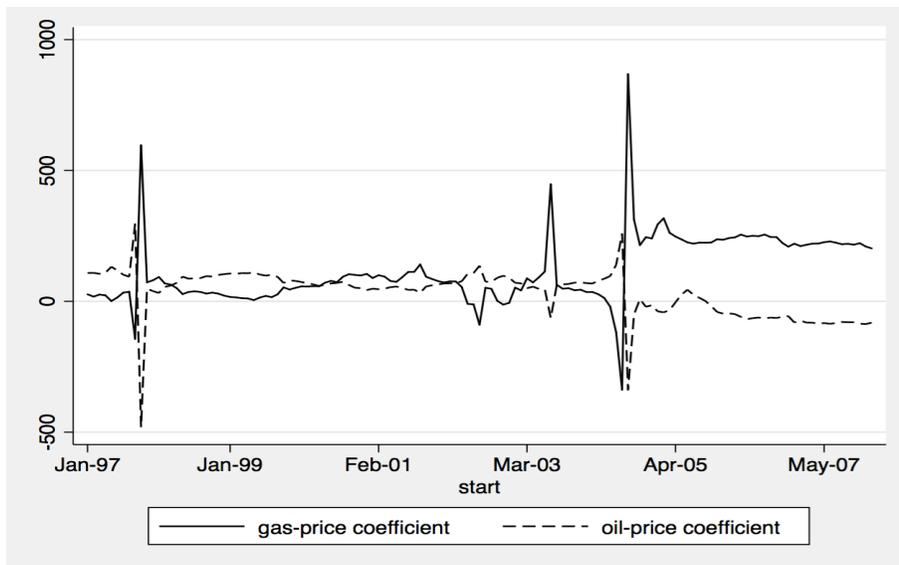


Figure 9: Coefficients from rolling oil rig cointegrating equation over sample (100 observation windows with sample starting in November 2004).

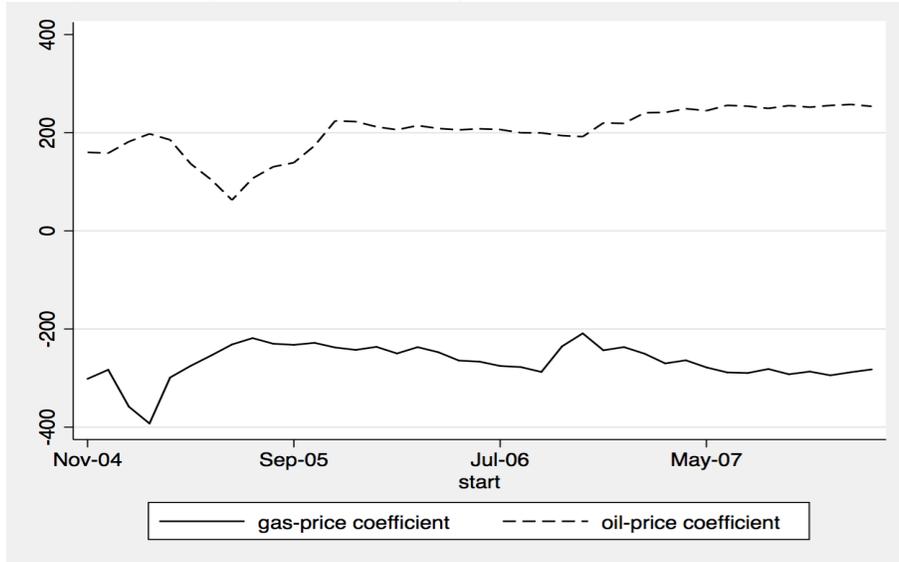


Figure 10: Coefficients from rolling gas rig cointegrating equation over sample (100 observation windows with sample starting in November 2004).

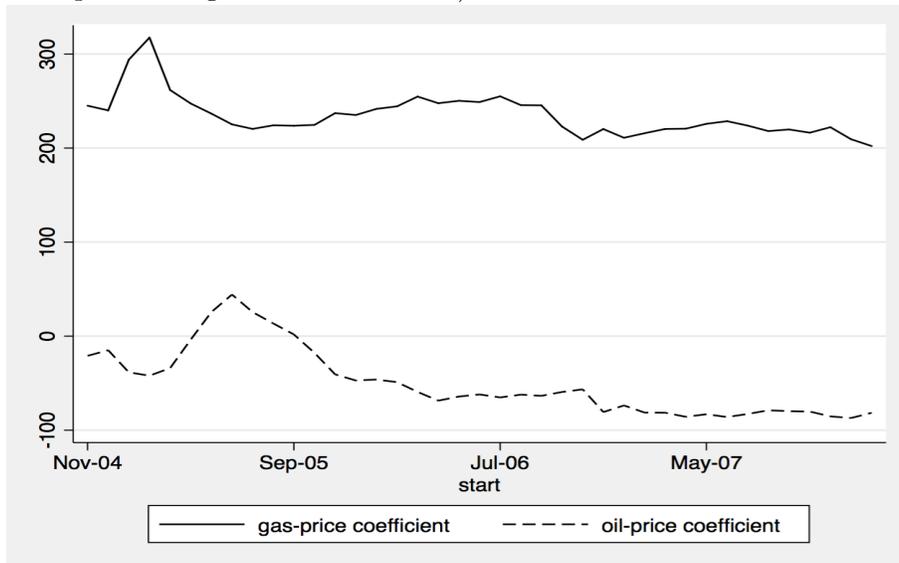


Figure 11: Coefficients from rolling oil rig 3SLS over sample (100 observation windows).

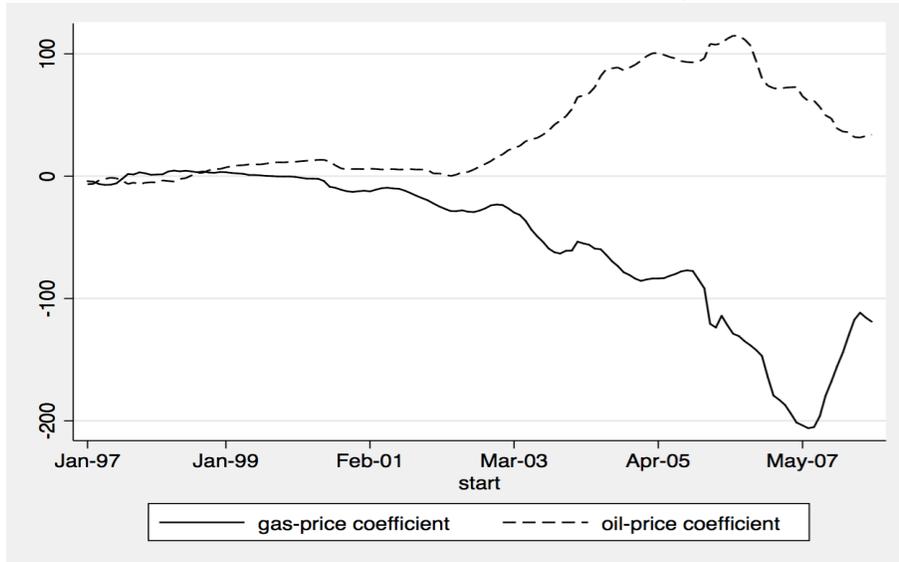
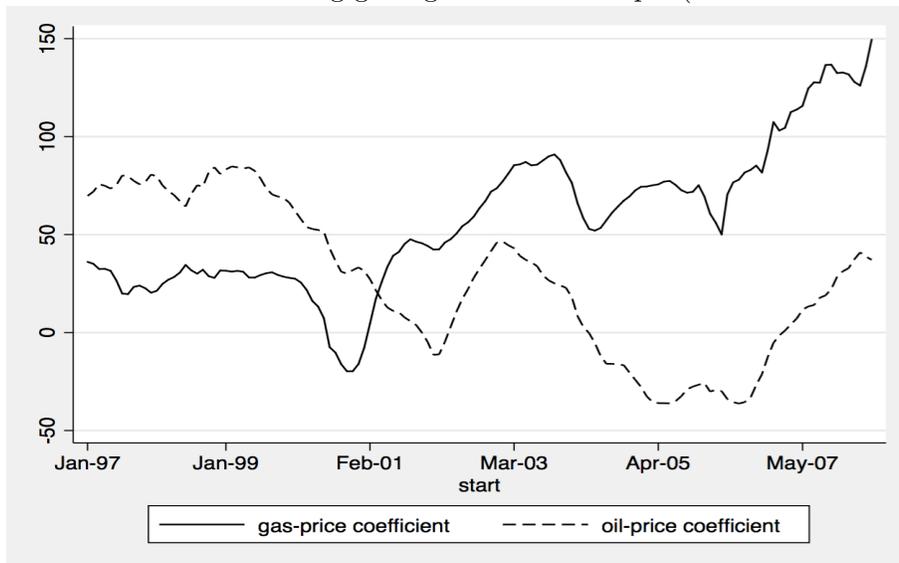


Figure 12: Coefficients from rolling gas rig 3SLS over sample (100 observation windows).



## Appendix

Table 2: Short-run parameters from VECM estimation. Lagged errors from the cointegrating vectors are  $Lce1$  for the oil equation  $Lce2$  for the gas equation.

	(1)	(2)	(3)	(4)
	$\Delta q_o$	$\Delta q_g$	$\Delta P_g$	$\Delta P_o$
$Lce1$	-0.0251*** (-3.43)	-0.022** (-2.93)	-0.0006* (23.28)	0.000 (0.04)
$Lce2$	-0.0099 (-1.24)	-0.046*** (-5.56)	-0.00007 (-0.23)	-0.00 (-0.01)
$\Delta q_{o_{t-1}}$	0.708*** (17.31)	0.120** (2.84)	0.0007 (0.45)	-0.002 (-1.20)
$\Delta q_{g_{t-1}}$	0.075 (1.73)	0.604*** (13.44)	-0.001 (-0.59)	-0.002 (-1.05)
$\Delta P_{g_{t-1}}$	1.319 (0.77)	-3.558* (-2.00)	-0.0274 (-0.40)	0.0318 (0.54)
$\Delta P_{o_{t-1}}$	7.392*** (3.63)	4.280* (2.02)	-0.0345 (-0.42)	0.327*** (4.64)
$N$	232	232	232	232

t statistics in parentheses

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$

Table 3: Alternative 3SLS specifications.

	(1)	(2)	(3)	(4)
	Preferred	W/out Price Forecast Errors	Only Price Forecast Errors	W/ Production & Stock Errors
$\mathbf{q}_o(t)$				
$P_g(t)$	-66.96*** (16.85)	-36.98 (52.98)	-55.56* (23.28)	-76.55*** (15.84)
$P_o(t)$	52.17*** (12.22)	141.3*** (30.18)	12.57 (18.23)	55.94*** (11.74)
Constant	333.9* (132.9)	-695.8* (350.6)	668.3*** (194.9)	345.9** (128.3)
$\mathbf{q}_g(t)$				
$P_g(t)$	64.78*** (16.12)	77.98 (42.83)	44.93* (18.18)	70.60*** (15.38)
$P_o(t)$	-1.415 (11.69)	-38.43 (24.39)	12.16 (14.23)	-2.717 (11.40)
Constant	484.0*** (127.1)	776.3** (283.4)	451.5** (152.2)	466.9*** (124.5)
$N$	229	234	229	229
$R^2$	0.57	0.05	0.30	0.59

Standard errors in parentheses

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$

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